

Supporting material II: MATALB code:

Main File:

```
global i
p0=0.02; %initial guess for CO2 partial pressure

options=optimset('algorithm','interior-point');

i=1;
[p,fval]=fmincon(@biomass_problem,p0,[],[],[],[],0,1,[],options);
b=zeros(1,6);
b(1)=-fval;
c(1,1)=p;

v=[p;p] ;
p0=v(:)';

for i=2:5

    n=2^(i-1);

    %set the lower and upper bound
    LB=zeros(1,n);
    UB=ones(1,n);

    %find the optimal CO2 profiles
    [p,fval]=fmincon(@biomass_problem,p0,[],[],[],[],LB,UB,[],options);

    b(i)=-fval; % record final biomass production in each loop
    for j=1:n
        c(j,i)=p(j);
        %record the CO2 pressures in each interval
        %pi is the CO2 pressure in the (i)th interval
    end

    %create new initial guesses for time intervals in the next loop
    v=[p;p] ;
    p0=v(:)';

    if abs((b(i)-b(i-1))/b(i-1))<0.0001, break, end;

end

load data X T;
%X is the biomass concentration and T is the timespan

%plot the results
subplot(2,2,[1,3]);
plot(T,X(:,1),'.');
xlabel('time/min');
ylabel('biomass concentration/ (mg/L)');
title('biomass');
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subplot(2,2,[2,4]);
plot(T,X(:,2),'.');
xlabel('time/min');
ylabel('CO_2 concentration in the algae culture/(umol/L)');
title('CO_2 concentration in the algae culture');

Function File 1:
function [err]=biomass_problem(p)

% this function serves to find the optimal CO2 profiles

global i

n=2^(i-1); % number of parts

tspan=0:0.5:17280/n;% timespan of the first interval
[T, X]=ode23s(@Flue_gas,tspan,[100,13],[],p(1));
%solve the equations with initial conditions that X(0)=100mg/L
%S(0)=13umol/L in the first interval

initial=X(end,:);
%the initial values for the next interval
%is the final values in current interval
X(end,:)=[];
T(end,:)=[];
% the final values are deleted otherwise they will appear both in the
final
% point in the current interval and initial point in the next interval

for m=2:n

    %obtain X and S in each interval
    tspan=17280/n*(m-1):0.5:17280/n*m;
    [t, x]=ode23s(@Flue_gas,tspan,initial,[],p(m));
    initial=x(end,:);

    x(end,:)=[];
    t(end,:)=[];

    X=[X;x];% put the results of each interval in order
    T=[T;t];

end

err=-X(end,1);
%the objective is to maximize the final biomass concentration

save data X T;
%save data for plotting

end

```

Function File 2:

```

function ydot=Flue_gas(t,y,p)

% p is the concentration of CO2 in the gas

```

```

%parameters
Ks=0.21;           %umol/L
Ki=10000;          %umol/L
umax=0.070/60;     %min^-1
KLa=17/60;         %min^-1
H=31.6/1000000;
%Henry constant(atm/M) 1000000 changes unit to mM/L
Y=100;             %mg(biomass)produced/umol(CO2)consumed
I0=45;             %micromole photons/m^2/s
K=14;              %micromole photons/m^2/s
A=0.0147;          %milligram/liter
kd=0.028/60;       %death rate
ydot=zeros(2,1);

I=I0/A/y(1)*(1-exp(-A*y(1))); %average light intensity

%ODE equations y(1) represents X; y(2) represents S;
ydot(1)=y(2)/(y(2)+y(2)^2/Ki+Ks)*I/(I+K)*umax*y(1)-kd*y(1);%equation 1
ydot(2)=KLa*(p/H-y(2))-Y*(ydot(1)+kd*y(1));%equation 2

end

```